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Massive 4D, $\mathcal{N} = 1$ superspin 1 & 3/2 multiplets and dualities

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Abstract: Lagrangians for several new off-shell 4D, $\mathcal{N} = 1$ supersymmetric descriptions of massive superspin-1 and superspin-3/2 multiplets are described. Taken together with the models previously constructed, there are now four off-shell formulations for the massive gravitino multiplet (superspin-1) and six off-shell formulations for the massive graviton multiplet (superspin-3/2). Duality transformations are derived which relate some of these dynamical systems.

Keywords: Supersymmetry and Duality, Supersymmetric Effective Theories.

Contents

1. Introduction

Different aspects of higher spin field theory in various dimensions attract considerable attention currently. First of all, higher spin fields and their possible interactions bring about numerous challenges for theoreticians. More importantly, massive higher spin states are known to be present in the spectra of the string and superstring theories. It is therefore quite natural to expect that, in a field theory limit, the superstring theory should reduce to a consistent interacting supersymmetric theory of higher spin fields.

In four space-time dimensions, lagrangian formulations for massive fields of arbitrary spin were constructed thirty years ago [1]. A few years later, the massive construction of [1] was used to derive lagrangian formulations for gauge massless fields of arbitrary spin [2]. Since then, there have been published hundreds of papers in which the results of [1, 2] were generalized, (BRST) reformulated, extended, quantized, and so forth. Here it is hardly possible to comment upon these heroic follow-up activities. We point out only several reviews [3] and some recent papers [4].

One of the interesting directions in higher spin field theory is the construction of manifestly supersymmetric extensions of the models given in [1, 2]. In the massless case, the problem has actually been solved in [5, 6] (see [7] for a review and [8] for generalizations). For each superspin $Y > 3/2$, these publications provide two dually equivalent off-shell realizations in 4D, $\mathcal{N} = 1$ superspace. At the component level, each of the two superspin-Y actions [5, 6] reduces to a sum of the spin-Y and spin-$(Y+1/2)$ actions [2] upon imposing a Wess-Zumino-type gauge and eliminating the auxiliary fields. On the mass shell, the only independent gauge-invariant field strengths in these models are exactly the higher spin on-shell field strengths first identified in "Superspace" [9]. As concerns the massive case, off-shell higher spin supermultiplets have never been constructed in complete generality.

In 4D, $\mathcal{N} = 1$ Poincaré supersymmetry, a massive multiplet of superspin Y describes four propagating fields with the same mass but different spins $s = (Y - 1/2, Y, Y, Y + 1/2)$, see, e.g., [7, 9] for reviews. The first attempts[1] to attack the problem of constructing free off-shell massive higher spin supermultiplets were undertaken in recent works [10]–[12] that were concerned with deriving off-shell realizations for the massive *gravitino multiplet* ($Y = 1$) and the massive *graviton multiplet* ($Y = 3/2$). This led to two $Y = 3/2$ formulations constructed in [10] and one $Y = 1$ formulation derived in [11]. The results of [10] were soon generalized [12] to produce a third $Y = 3/2$ formulation.

In the present letter, we continue the research started in [10, 11] and derive two new off-shell realizations for the massive gravitino multiplet, and three new off-shell realizations for the massive graviton multiplet. Altogether, there now occur four massive $Y = 1$ models (in conjunction with the massive $Y = 1$ model constructed by Ogievetsky and Sokatchev years ago [14]) and six massive $Y = 3/2$ models. We further demonstrate that these realizations are related to each other by duality transformations similar to those which relate massive tensor and vector multiplets, see [15] and references therein.

It is interesting to compare the massive and massless results in the case of the $Y = 3/2$ multiplet. In the massless case, there are three building blocks to construct *minimal* [2] linearized supergravity [16]. They correspond to (i) old minimal supergravity (see [7, 9] for reviews); (ii) new minimal supergravity (see [7, 9] for reviews); (iii) the novel formulation derived in [10]. These off-shell $(3/2, 2)$ supermultiplets, which comprise all the supergravity multiplets with $12 + 12$ degrees of freedom, will be called type I, type II and type III supergravity multiplets[3] in what follows, in order to avoid the use of unwieldy terms like "new new minimal" or "very new" supergravity. As is demonstrated below, each of the massless type I–III formulations admits a massive extension, and the latter turns out to possess a nontrivial dual. As a result, we have now demonstrated that there occur *at least* six off-shell distinct massive $Y = 3/2$ minimal realizations.

This paper is organized as follows. In section 2, we derive two new (dually equivalent) formulations for the massive gravitino multiplet. They turn out to be massive extensions of the two standard off-shell formulations for the massless spin $(1, 3/2)$ supermultiplet discovered respectively in [17]–[19] and [20]. In section 3, we derive three new formulations for the massive graviton multiplet. Duality transformations are also worked out that relate all the massive $Y = 3/2$ models. A brief summary of the results obtained is given in section 4. The paper is concluded by two technical appendices. Our superspace conventions mostly follow [7] except the following two from [9]: (i) the symmetrization of n indices does not involve a factor of $(1/n!)$; (ii) given a four-vector v_a, we define $v_{\underline{a}} \equiv v_{\alpha\dot{\alpha}} = (\sigma^a)_{\alpha\dot{\alpha}} v_a$.

[1]Some preliminary results were also obtained in [13].

[2]To our knowledge, no investigations have occured for the possible existence of a massive *non-minimal* $Y = 3/2$ theory.

[3]In the case of type III supergravity, a nonlinear formulation is still unknown.

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2. Massive gravitino multiplets

We start by recalling the off-shell formulation for massless (matter) gravitino multiplet introduced first in [17, 18] at the component level and then formulated in [19] in terms of superfields (see also [21]). The action derived in [19] is

$$
\begin{aligned}
S_{(1,\frac{3}{2})}[\Psi, V] &= \hat{S}[\Psi] + \int d^8 z \left\{ \Psi^\alpha W_\alpha + \overline{\Psi}_{\dot\alpha} \overline{W}^{\dot\alpha} \right\} - \frac{1}{4} \int d^6 z \, W^\alpha W_\alpha \,, \\
W_\alpha &= -\frac{1}{4} \overline{D}^2 D^\alpha V \,,
\end{aligned}
\tag{2.1}
$$

where

$$
\hat{S}[\Psi] = \int d^8 z \left\{ D^\alpha \overline{\Psi}^{\dot\alpha} \overline{D}_{\dot\alpha} \Psi_\alpha - \frac{1}{4} \overline{D}^{\dot\alpha} \Psi^\alpha \overline{D}_{\dot\alpha} \Psi_\alpha - \frac{1}{4} D_\alpha \overline{\Psi}_{\dot\alpha} D^\alpha \overline{\Psi}^{\dot\alpha} \right\} . \tag{2.2}
$$

This massless $Y = 1$ model is actually of some interest in the context of higher spin field theory. As mentioned in the introduction, there exist two dually equivalent gauge superfield formulations (called longitudinal and transverse) [6] for each massless *integer* $Y \geq 1$, see [7] for a review. The longitudinal series[4] terminates, at $Y = 1$, exactly at the action (2.1).

To describe a massive gravitino multiplet, we introduce an action $S = S_{(1,\frac{3}{2})}[\Psi, V] + S_m[\Psi, V]$, where $S_m[\Psi, V]$ stands for the mass term

$$
S_m[\Psi, V] = m \int d^8 z \left\{ \Psi^2 + \overline{\Psi}^2 + \alpha m V^2 + V \left(\beta D^\alpha \Psi_\alpha + \beta^* \overline{D}_{\dot\alpha} \overline{\Psi}^{\dot\alpha} \right) \right\} , \tag{2.3}
$$

where α and β are respectively real and complex parameters. These parameters should be fixed by the requirement that the equations of motion be equivalent to the constraints

$$
i \partial_{\underline{a}} \overline{\Psi}^{\dot\alpha} + m \Psi_\alpha = 0 \,, \qquad D^\alpha \Psi_\alpha = 0 \,, \qquad \overline{D}^2 \Psi_\alpha = 0 \,, \tag{2.4}
$$

required to describe an irreducible on-shell multiplet with $Y = 1$, see [7, 11]. In the space of spinor superfields obeying the Klein-Gordon equation, $(\Box - m^2)\Psi_\alpha = 0$, the second and third constraints in (2.4) are known to select the $Y = 1$ subspace [7] (see also [22]). Without imposing additional constraints (such as the first one in (2.4)), the superfields Ψ_α and $\overline{\Psi}_{\dot\alpha}$ describe two massive $Y = 1$ representations. Generally, an irreducible representation emerges if these superfields are also subject to a reality condition of the form

$$
\partial_{\underline{a}} \overline{\Psi}^{\dot\alpha} + m \, e^{i\varphi} \, \Psi_\alpha = 0 \,, \qquad |e^{i\varphi}| = 1 \,, \tag{2.5}
$$

where φ a constant real parameter. As is obvious, the latter constraint implies the Klein-Gordon equation. Applying a phase transformation to Ψ_α, allows us to make the choice $e^{i\varphi} = -i$ corresponding to the Dirac equation.

The equations of motion corresponding to $S = S_{(1,\frac{3}{2})}[\Psi, V] + S_m[\Psi, V]$ are:

$$
-\overline{D}_{\dot\alpha} D_\alpha \overline{\Psi}^{\dot\alpha} + \frac{1}{2} \overline{D}^2 \Psi_\alpha + 2m \Psi_\alpha + W_\alpha - \beta m D_\alpha V = 0 \,, \tag{2.6}
$$

$$
\frac{1}{2} D^\alpha W_\alpha + \left(\frac{1}{4} D^\alpha \overline{D}^2 \Psi_\alpha + \beta m D^\alpha \Psi_\alpha + \text{c.c.} \right) + 2\alpha m^2 V = 0 \,. \tag{2.7}
$$

[4] The transverse series terminates at a non-minimal gauge formulation for the massless gravitino multiplet realized in terms of an unconstrained real scalar V and Majorana γ-traceless spin-vector $\boldsymbol{\Psi}_a = (\Psi_{a\beta}, \overline{\Psi}_a{}^{\dot\beta})$, with $\gamma^a \boldsymbol{\Psi}_a = 0$.

Multiplying (2.6) and (2.7) by $\overline{D}^2$ yields:

$$\overline{D}^2\Psi_\alpha = -2\beta W_\alpha\,, \qquad \overline{D}^2 D^\alpha \Psi_\alpha = -2\frac{\alpha}{\beta}\, m\overline{D}^2 V\,. \tag{2.8}$$

Next, substituting these relations into the contraction of D^α on (2.6) leads to:

$$mD^\alpha\Psi_\alpha = \frac{1}{2}(\beta+\beta^*-1)D^\alpha W_\alpha + \frac{\beta}{2}\left(1+\frac{\alpha}{|\beta|^2}\right) mD^2V\,. \tag{2.9}$$

Substitute these three results into (2.7) gives

$$\frac{1}{2}(1-\beta-\beta^*)^2 D^\alpha W_\alpha + \frac{1}{2}m\left(1+\frac{\alpha}{|\beta|^2}\right)[\beta^2 D^2 + (\beta^*)^2\overline{D}^2]V + 2\alpha m^2 V = 0\,. \tag{2.10}$$

This equation implies that V is auxiliary, $V=0$, if

$$\beta+\beta^* = 1\,, \qquad \alpha = -|\beta|^2\,. \tag{2.11}$$

Then, the mass-shell conditions (2.4) also follow.[5]

The final action takes the form:

$$S[\Psi,V] = \hat{S}[\Psi] + \int d^8z \left\{\Psi\, W + \overline{\Psi}\,\overline{W}\right\} - \frac{1}{4}\int d^6z\, W^2 + \\ + m\int d^8z\,\left\{\Psi^2 + \overline{\Psi}^2 - |\beta|^2 mV^2 + V\Big(\beta D\,\Psi + \beta^*\overline{D}\,\overline{\Psi}\Big)\right\}, \tag{2.12}$$

where $\beta+\beta^*=1$. A superfield redefinition of the form $\Psi_\alpha \to \Psi_\alpha + \delta\,\overline{D}^2\Psi_\alpha$ can be used to change some coefficients in the action.

The lagrangian constructed turns out to possess a dual formulation. For simplicity, we choose $\beta = 1/2$ in (2.12). Let us consider, following [15], the "first-order" action

$$S_{\rm Aux} = \hat{S}[\Psi] + \int d^8z\left\{m(\Psi^2+\overline{\Psi}^2) + \Psi\, W + \overline{\Psi}\,\overline{W} - \frac{m^2}{4}V^2 + \frac{m}{2}V(D\,\Psi + \overline{D}\,\overline{\Psi})\right\} + \\ + \frac{1}{2}\left\{m\int d^6z\,\eta^\alpha\Big(W_\alpha + \frac{1}{4}\overline{D}^2 D^\alpha V\Big) - \frac{1}{4}\int d^6z\, W^2 \ + \ \text{c.c.}\right\}. \tag{2.13}$$

Here W_α and η_α are unconstrained chiral spinor superfield, and there is no relationship between V and W_α. Varying S_{Aux} with respect to η_α brings us back to (2.12). On the other hand, if we vary S_{Aux} with respect to V and W_α and eliminate these superfields, we then arrive at the following action:

$$\tilde{S} = \hat{S}[\Psi] + \int d^8z\left\{m(\Psi^2+\overline{\Psi}^2) + \frac{1}{4}\Big(D(\Psi+\eta) + \overline{D}(\overline{\Psi}+\overline{\eta})\Big)^2\right\} + \\ + \frac{1}{8}\left\{\int d^6z\,\Big(2m\eta - \overline{D}^2\Psi\Big)^2 \ + \ \text{c.c.}\right\}. \tag{2.14}$$

[5] One can consider more general actions in which the term $m(\Psi^2+\overline{\Psi}^2)$ in (2.3) is replaced by $(\mu\,\Psi^2 + \mu^*\,\overline{\Psi}^2)$, with μ a complex mass parameter, $|\mu| = m$. Then, the first equation in (2.11) turns into $\beta/\mu + (\beta/\mu)^* = 1/m$.

Implementing here the shift

$$\Psi_\alpha \ \to \ \Psi_\alpha - \eta_\alpha\,, \tag{2.15}$$

brings the action to the form

$$\begin{aligned}\tilde{S} \,=\, \hat{S}[\Psi] + \frac{1}{4}\int d^8z \left(D\,\Psi + \overline{D}\,\overline{\Psi}\right)^2 - \frac{1}{2}\int d^8z \left\{\Psi^\alpha \overline{D}^2 \Psi_\alpha + \overline{\Psi}_{\dot\alpha} D^2 \overline{\Psi}^{\dot\alpha}\right\} + \\ + m\int d^8z \left\{\Psi^2 + \overline{\Psi}^2\right\} + \frac{m^2}{2}\left\{\int d^6z\,\eta^2 \ + \ \text{c.c.}\right\}.\end{aligned} \tag{2.16}$$

As is seen, the chiral spinor superfield η_α has completely decoupled! Therefore, the dynamical system obtained is equivalent to the following theory

$$\begin{aligned}S[\Psi] \,=\, \hat{S}[\Psi] + \frac{1}{4}\int d^8z \left(D\,\Psi + \overline{D}\,\overline{\Psi}\right)^2 - \frac{1}{2}\int d^8z \left\{\Psi^\alpha \overline{D}^2 \Psi_\alpha + \overline{\Psi}_{\dot\alpha} D^2 \overline{\Psi}^{\dot\alpha}\right\} + \\ + m\int d^8z\ \left\{\Psi^2 + \overline{\Psi}^2\right\},\end{aligned} \tag{2.17}$$

formulated solely in terms of the unconstrained spinor Ψ_α and it conjugate. Applying the phase transformation $\Psi_a \to i\,\Psi_\alpha$, it is seen that the action obtained is actually equivalent to

$$S[\Psi] = \hat{S}[\Psi] - \frac{1}{4}\int d^8z \left(D\,\Psi - \overline{D}\,\overline{\Psi}\right)^2 + m\int d^8z\ \left\{\Psi^2 + \overline{\Psi}^2\right\}. \tag{2.18}$$

It is interesting to compare (2.18) with the action for massive $Y = 1$ multiplet obtained by Ogievetsky and Sokatchev [14]. Their model is also formulated solely in terms of a spinor superfield. The corresponding action[6] is

$$S_{OS}[\Psi] = \hat{S}[\Psi] + \frac{1}{4}\int d^8z \left(D\Psi + \overline{D\Psi}\right)^2 + i\,m\int d^8z \left(\Psi^2 - \overline{\Psi}^2\right), \tag{2.19}$$

see appendix A for its derivation.[7] The actions (2.18) and (2.19) look similar, although it does not seem possible to transform one to the other off the mass shell.

In fact, the model (2.14), which is equivalent to (2.18), can be treated as a massive extension of the Ogievetsky-Sokatchev model for massless gravitino multiplet [20]. Indeed, implementing in (2.14) the shift

$$\Psi_\alpha \ \to \Psi_\alpha + \frac{i}{2m}\overline{D}^2\Psi_\alpha\,, \qquad \eta_\alpha \ \to \eta_\alpha - \frac{i}{2m}\overline{D}^2\Psi_\alpha\,, \tag{2.20}$$

which leaves $\hat{S}[\Psi]$ invariant, we end up with

$$\begin{aligned}S[\Psi,\eta] \,=\, S_{(1,\frac{3}{2})}[\Psi,G] + m\int d^8z \left\{\Psi^2 + \overline{\Psi}^2 + 2(1+i)\Psi\eta + 2(1-i)\overline{\Psi}\overline{\eta}\right\} + \\ + \frac{m^2}{2}\left\{\int d^6z\,\eta^2 \ + \ \text{c.c.}\right\},\end{aligned} \tag{2.21}$$

[6]Setting $m = 0$ in (2.19) gives the model for massless gravitino multiplet discovered in [20].

[7]It was argued in [11] that there are no lagrangian formulations for massive superspin-1 multiplet solely in terms of an unconstrained spinor superfield and its conjugate. The "proof" given in [11] is incorrect, as shown by the two counter-examples (2.18) and (2.19).

where

$$
\begin{aligned}
S_{(1,\frac{3}{2})}[\Psi, G] &= \hat{S}[\Psi] + \int d^8z \, \Big(G + \frac{1}{2}(D\,\Psi + \overline{D}\,\overline{\Psi}) \Big)^2 , \\
G &= \frac{1}{2}(D^\alpha \eta_\alpha + \overline{D}_{\dot\alpha} \bar\eta^{\dot\alpha}) \, .
\end{aligned} \tag{2.22}
$$

Here G is the linear superfield, $D^2 G = \overline{D}^2 G = 0$, associated with the chiral spinor η_α and its conjugate. The action $S_{(1,\frac{3}{2})}[\Psi, G]$ corresponds to the Ogievetsky-Sokatchev formulation for massless gravitino multiplet [20] as presented in [7].

Before concluding this section, it is worth recalling one more possibility to describe the massless gravitino multiplet [7, 19]

$$
S_{(1,\frac{3}{2})}[\Psi, \Phi] = \hat{S}[\Psi] - \frac{1}{2} \int d^8z \, \Big\{ \overline{\Phi}\Phi + (\Phi + \overline{\Phi})(D\,\Psi + \overline{D}\,\overline{\Psi}) \Big\} , \tag{2.23}
$$

with Φ a chiral scalar, $\overline{D}_{\dot\alpha}\Phi = 0$. The actions (2.1) and (2.23) can be shown to correspond to different partial gauge fixings in the mother theory

$$
\begin{aligned}
S_{(1,\frac{3}{2})}[\Psi, V, \Phi] &= \hat{S}[\Psi] + \int d^8z \, \Big\{ \Psi\, W + \overline{\Psi}\,\overline{W} \Big\} - \frac{1}{4} \int d^6z \, W^2 - \\
&\quad - \frac{1}{2} \int d^8z \, \Big\{ \overline{\Phi}\Phi + (\Phi + \overline{\Phi})(D\,\Psi + \overline{D}\,\overline{\Psi}) \Big\} ,
\end{aligned} \tag{2.24}
$$

possessing a huge gauge freedom, see [7, 19] for more details. The massive extension of (2.23) was derived in [11] and the corresponding action is

$$
S[\Psi, \Phi] = S_{(1,\frac{3}{2})}[\Psi, \Phi] + m \int d^8z \, (\Psi^2 + \overline{\Psi}^2) - \frac{m}{4} \Big\{ \int d^6z \, \Phi^2 + \text{c.c.} \Big\} . \tag{2.25}
$$

Unlike its massless limit, this theory does not seem to admit a nice dual formulation.

3. Massive graviton multiplets

The massive $Y = 3/2$ multiplet (or massive graviton multiplet) can be realized in terms of a real (axial) vector superfield H_a obeying the equations [7, 10, 22]

$$
(\Box - m^2) H_a = 0 \, , \qquad D^\alpha H_{\underline{a}} = 0 \, , \qquad \overline{D}^{\dot\alpha} H_{\underline{a}} = 0 \quad \longrightarrow \quad \partial^{\underline{a}} H_{\underline{a}} = 0 \, . \tag{3.1}
$$

We are interested in classifying those supersymmetric theories which generate these equations as the equations of motion.

In what follows, we will use a set of superprojectors [23] for the real vector superfield $H_{\underline{a}}$:

$$
\begin{aligned}
(\Pi^T_1) H_{\underline{a}} &:= \frac{1}{32} \Box^{-2} \partial_{\dot\alpha}{}^\beta \{\overline{D}^2, D^2\} \partial_{(\alpha}{}^{\dot\beta} H_{\beta)\dot\beta} \, , &(3.2)\\
(\Pi^T_{1/2}) H_{\underline{a}} &:= \frac{1}{8\cdot 3!} \Box^{-2} \partial_{\dot\alpha}{}^\beta D_{(\alpha} \overline{D}^2 D^\gamma (\partial_{\beta)}{}^{\dot\beta} H_{\gamma\dot\beta} + \partial_{|\gamma|}{}^{\dot\beta} H_{\beta)\dot\beta}) \, , &(3.3)\\
(\Pi^T_{3/2}) H_{\underline{a}} &:= -\frac{1}{8\cdot 3!} \Box^{-2} \partial_{\dot\alpha}{}^\beta D^\gamma \overline{D}^2 D_{(\gamma} \partial_\alpha{}^{\dot\beta} H_{\beta)\dot\beta} \, , &(3.4)\\
(\Pi^L_0) H_{\underline{a}} &:= -\frac{1}{32} \partial_{\underline{a}} \Box^{-2} \{\overline{D}^2, D^2\} \partial^{\underline{c}} H_{\underline{c}} \, , &(3.5)\\
(\Pi^L_{1/2}) H_{\underline{a}} &:= \frac{1}{16} \partial_{\underline{a}} \Box^{-2} D^\beta \overline{D}^2 D_\beta \partial^{\underline{c}} H_{\underline{c}} \, . &(3.6)
\end{aligned}
$$

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In terms of the superprojectors introduced, we have [16]

$$D^\gamma \overline{D}^2 D_\gamma H_{\underline{a}} = -8\Box \left(\Pi^L_{1/2} + \Pi^T_{1/2} + \Pi^T_{3/2}\right) H_{\underline{a}}\,, \tag{3.7}$$

$$\partial_{\underline{a}}\, \partial^{\underline{b}} H_{\underline{b}} = -2\Box \left(\Pi^L_0 + \Pi^L_{1/2}\right) H_{\underline{a}}\,, \tag{3.8}$$

$$[D_\alpha, \overline{D}_{\dot\alpha}] \left[D_\beta, \overline{D}_{\dot\beta}\right] H^{\underline{b}} = \Box \left(8\Pi^L_0 - 24\Pi^T_{1/2}\right) H_{\underline{a}}\,. \tag{3.9}$$

3.1 Massive extensions of type I supergravity

Consider the off-shell massive supergravity multiplet derived in [12]

$$S^{(\mathrm{IA})}[H,P] = S^{(\mathrm{I})}[H,\Sigma] - \frac{1}{2} m^2 \int d^8 z\, \left\{ H^{\underline{a}} H_{\underline{a}} - \frac{9}{2} P^2 \right\}, \tag{3.10}$$

where the massless part of the action takes the form

$$\begin{aligned} S^{(\mathrm{I})}[H,\Sigma] &= \int d^8 z\, \left\{ H^{\underline{a}} \Box \left(-\frac{1}{3}\Pi^L_0 + \frac{1}{2}\Pi^T_{3/2}\right) H_{\underline{a}} - i(\Sigma - \overline{\Sigma})\partial^{\underline{a}} H_{\underline{a}} - 3\overline{\Sigma}\Sigma \right\}, \\ \Sigma &= -\frac{1}{4}\overline{D}^2 P\,, \qquad \overline{P} = P\,, \end{aligned} \tag{3.11}$$

and this corresponds to a linearized form of type I (old minimal) supergravity that has only appeared in the research literature [24]. It has not been discussed in textbooks such as [7, 9]. The distinctive feature *unique* to this theory is that its set of auxiliary fields contains one axial vector, one scalar and one three-form (S, $C_{\underline{a}\,\underline{b}\,\underline{c}}$, $A_{\underline{a}}$). Interestingly enough and to our knowledge, there has *never* been constructed a massive theory that contains the standard auxiliary fields of minimal supergravity (S, P, $A_{\underline{a}}$). This fact may be of some yet-to-be understood significance.

The theory with action $S^{(\mathrm{IA})}[H,P]$ turns out to possess a dual formulation. Let us introduce the "first-order" action

$$\begin{aligned} S_{\mathrm{Aux}} = \int d^8 z\, \Big\{ & H^{\underline{a}} \Box \left(-\frac{1}{3}\Pi^L_0 + \frac{1}{2}\Pi^T_{3/2}\right) H_{\underline{a}} - \frac{1}{2} m^2 H^{\underline{a}} H_{\underline{a}} - U \partial^{\underline{a}} H_{\underline{a}} - \\ & - \frac{3}{2} U^2 + \frac{9}{4} m^2 P^2 + 3mV \left(U + \frac{1}{4}\overline{D}^2 P + \frac{1}{4} D^2 P \right) \Big\}, \end{aligned} \tag{3.12}$$

where U and V are real unconstrained superfields. Varying V brings us back to (3.10). On the other hand, we can eliminate U and P using their equations of motion. With the aid of (3.8), this gives

$$\begin{aligned} S^{(\mathrm{IB})}[H,P] = \int d^8 z\, \Big\{ & H^{\underline{a}} \Box \left(\frac{1}{3}\Pi^L_{1/2} + \frac{1}{2}\Pi^T_{3/2}\right) H_{\underline{a}} - \frac{1}{2} m^2 H^{\underline{a}} H_{\underline{a}} - \\ & - \frac{1}{16} V \{\overline{D}^2, D^2\} V - mV \partial^{\underline{a}} H_{\underline{a}} + \frac{3}{2} m^2 V^2 \Big\}. \end{aligned} \tag{3.13}$$

This is one of the two formulations for the massive $Y = 3/2$ multiplet constructed in [10].

3.2 Massive extensions of type II supergravity

Let us now turn to type II (or new minimal) supergravity. Its linearized action is

$$\begin{aligned} S^{(\mathrm{II})}[H,\mathcal{U}] &= \int d^8z\, \left\{ H^{\underline{a}}\Box\left(-\Pi^T_{1/2}+\frac{1}{2}\Pi^T_{3/2}\right)H_{\underline{a}}+\frac{1}{2}\mathcal{U}[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}}+\frac{3}{2}\mathcal{U}^2\right\}\,, \\ \mathcal{U} &= D^\alpha\chi_\alpha+\overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\,, \qquad \overline{D}_{\dot\alpha}\chi_\alpha=0\,, \end{aligned} \tag{3.14}$$

with χ_α an unconstrained chiral spinor. It possesses a unique massive extension

$$S^{(\mathrm{IIA})}[H,\chi]=S^{(\mathrm{II})}[H,\mathcal{U}]-\frac{1}{2}m^2\int d^8z\, H^{\underline{a}}H_{\underline{a}}+3m^2\left\{\int d^6z\,\chi^2+\mathrm{c.c.}\right\} \tag{3.15}$$

which is derived in appendix B.

The theory (3.15) admits a dual formulation. Let us consider the following "first-order" action

$$\begin{aligned} S_{\mathrm{Aux}} = \int d^8z\, &\left\{ H^{\underline{a}}\Box\left(-\Pi^T_{1/2}+\frac{1}{2}\Pi^T_{3/2}\right)H_{\underline{a}}-\frac{1}{2}m^2H^{\underline{a}}H_{\underline{a}}+\frac{1}{2}\mathcal{U}[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}}+\frac{3}{2}\mathcal{U}^2- \right.\\ &\left. -6mV\left(\mathcal{U}-D^\alpha\chi_\alpha-\overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\right)\right\}+3m^2\left\{\int d^6z\,\chi^\alpha\chi_\alpha+\mathrm{c.c.}\right\}, \end{aligned} \tag{3.16}$$

in which $\mathcal{U}$ and V are real unconstrained superfields. Varying V gives the original action (3.15). On the other hand, we can eliminate the independent real scalar $\mathcal{U}$ and chiral spinor χ_α variables using their equations of motion. With the aid of (3.9) this gives

$$\begin{aligned} S^{(\mathrm{IIB})}[H,V] = \int d^8z\, &\left\{ H^{\underline{a}}\Box\left(-\frac{1}{3}\Pi^L_0+\frac{1}{2}\Pi^T_{3/2}\right)H_{\underline{a}}-\frac{1}{2}m^2H^{\underline{a}}H_{\underline{a}}+ \right.\\ &\left. +mV[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}}-6m^2V^2\right\}-6\int d^6z\,W^\alpha W_\alpha\,, \end{aligned} \tag{3.17}$$

where W_α is the vector multiplet field strength defined in (2.1). The obtained action (3.17) constitutes a new formulation for massive supergravity multiplet.

3.3 Massive extensions of type III supergravity

Let us now turn to linearized type III supergravity [10]

$$\begin{aligned} S^{(\mathrm{III})}[H,\mathcal{U}] &= \int d^8z\, \left\{ H^{\underline{a}}\Box\left(\frac{1}{3}\Pi^L_{1/2}+\frac{1}{2}\Pi^T_{3/2}\right)H_{\underline{a}}+\mathcal{U}\partial_{\underline{a}}H^{\underline{a}}+\frac{3}{2}\mathcal{U}^2\right\}, \\ \mathcal{U} &= D^\alpha\chi_\alpha+\overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\,, \qquad \overline{D}_{\dot\alpha}\chi_\alpha=0\,, \end{aligned} \tag{3.18}$$

with χ_α an unconstrained chiral spinor. It possesses a unique massive extension

$$S^{(\mathrm{IIIA})}[H,\chi]=S^{(\mathrm{III})}[H,\mathcal{U}]-\frac{1}{2}m^2\int d^8z\, H^{\underline{a}}H_{\underline{a}}-9m^2\left\{\int d^6z\,\chi^2+\mathrm{c.c.}\right\}, \tag{3.19}$$

and its derivation is very similar to that of (3.15) given in appendix B.

Similarly to the type II case considered earlier, the theory (3.19) admits a dual formulation. Let us introduce the "first-order" action

$$S_{\rm Aux} = \int d^8z \left\{ H^{\underline{a}}\Box \left(\frac{1}{3}\Pi^L_{1/2} + \frac{1}{2}\Pi^T_{3/2}\right) H_{\underline{a}} - \frac{1}{2}m^2 H^{\underline{a}} H_{\underline{a}} + \mathcal{U}\partial_{\underline{a}} H^{\underline{a}} + \frac{3}{2}\mathcal{U}^2 + \right.$$
$$\left. + 3mV \left(\mathcal{U} - D^\alpha \chi_\alpha - \bar{D}_{\dot\alpha}\bar{\chi}^{\dot\alpha}\right)\right\} - 9m^2 \left\{\int d^6z\, \chi^\alpha\chi_\alpha + {\rm c.c.}\right\}, \quad (3.20)$$

in which $\mathcal{U}$ and V are real unconstrained superfields. Varying V gives the original action (3.19). On the other hand, we can eliminate the independent real scalar $\mathcal{U}$ and chiral spinor χ_α variables using their equations of motion. With the aid of (3.8) this gives

$$S^{(\rm IIIB)}[H,V] = \int d^8z \left\{ H^{\underline{a}}\Box \left(-\frac{1}{3}\Pi^L_0 + \frac{1}{2}\Pi^T_{3/2}\right) H_{\underline{a}} - \frac{1}{2}m^2 H^{\underline{a}} H_{\underline{a}} - \right.$$
$$\left. - mV\partial_{\underline{a}} H^{\underline{a}} - \frac{3}{2}m^2V^2\right\} + \frac{1}{2}\int d^6z\, W^\alpha W_\alpha\,, \quad (3.21)$$

with the vector multiplet field strength W_α defined in eq. (2.1). This is one of the two formulations for the massive $Y = 3/2$ multiplet constructed in [10]. The other formulation is given by the action (3.13).

4. Summary

We have formulated new free superfield dynamical theories for massive multiplets of superspin $Y = 1$ and $Y = 3/2$. We have shown that these new theories are dually equivalent to the theories with corresponding superspin given previously in the literature [10]–[12]. Although the theories with a fixed and specific value of Y are on-shell equivalent, they differ from one another by distinctive sets of auxiliary superfields (see discussion of this point in [10]). The existence of their varied and distinctive off-shell structures together with their on-shell equivalence comes somewhat as a surprise.

This surprise suggests that there is much remaining work to be done in order to understand and classify the distinct off-shell representations for all multiplets with higher values of Y in both the massless and massive cases. Our results raise many questions. For example, for a fixed value of Y what massless off-shell representations possess massive extensions? How does the number of such duality related formulations depend on the value of Y? Are there even more off-shell possibilities for the massless theories uncovered in the works of [5, 6]? Another obvious question relates to the results demonstrated in the second work of [8]. In this past work, it was shown that there is a natural way to combine 4D, $\mathcal{N} = 1$ massless higher spin supermultiplets into 4D, $\mathcal{N} = 2$ massless higher spin supermultiplets. Therefore, we are led to expect that it should be possible to combine 4D, $\mathcal{N} = 1$ massive higher spin supermultiplets into 4D, $\mathcal{N} = 2$ massive higher spin supermultiplets. As we presently only possess *four* $Y = 1$ and *six* $Y = 3/2$ 4D, $\mathcal{N} = 1$ supermultiplets, the extension to 4D, $\mathcal{N} = 2$ supersymmetry promises to be an interesting study for the future.

All of these questions bring to the fore the need for a comprehensive understanding of the role of duality for arbitrary Y supersymmetric representations, of both the massless and massive varieties. In turn this raises the even more daunting specter of understanding the role of duality within the context of superstring/M-theory. To our knowledge the first time the question was raised about the possibility of dually related superstrings was in 1985 [25] and there the question concerns on-shell dually related theories. So for both on-shell and off-shell theories we lack a complete understanding of duality. The most successful descriptions of superstrings are of the type pioneered by Berkovits (see [13] and references therein). As presently formulated, there is no sign of duality in that formalism. So does the superstring uniquely pick out representations among the many dual varieties suggested by our work?

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A. Derivation of (2.19)

Let us start with the action

$$S[\Psi] = \hat{S}[\Psi] + \frac{1}{4}\int d^8 z\left(D\,\Psi + \overline{D}\,\overline{\Psi}\right)^2 + \int d^8 z\left(\mu\Psi^2 + \mu^*\,\overline{\Psi}^2\right), \tag{A.1}$$

where the functional $\hat{S}[\Psi]$ is defined in (2.2), and μ is a complex mass parameter to be specified later. The action (A.1) with $\mu = 0$ describes the Ogievetsky-Sokatchev model for the massless gravitino multiplet [20]. We are going to analyze whether this action with $\mu \neq 0$ can be used to consistently describe the massive gravitino multiplet dynamics. The equation of motion for Ψ^α is

$$-\overline{D}_{\dot\alpha} D_\alpha \overline{\Psi}^{\dot\alpha} + \frac{1}{2}\overline{D}^2\Psi_\alpha - \frac{1}{2}D_\alpha(D\,\Psi + \overline{D}\,\overline{\Psi}) + 2\mu\,\Psi_\alpha = 0\,. \tag{A.2}$$

It implies

$$-\frac{1}{4}\overline{D}^2 D_\alpha(D\,\Psi + \overline{D}\,\overline{\Psi}) + \mu\,\overline{D}^2\Psi_\alpha = 0\,, \tag{A.3}$$

and therefore

$$\begin{aligned} 0 &= -\frac{1}{4}D^\alpha\overline{D}^2 D_\alpha(D\,\Psi + \overline{D}\,\overline{\Psi}) + \mu\,D^\alpha\overline{D}^2\Psi_\alpha & \text{(A.4)}\\ &= -\frac{1}{4}D^\alpha\overline{D}^2 D_\alpha(D\,\Psi + \overline{D}\,\overline{\Psi}) + \mu\,\overline{D}^2(D\,\Psi + \overline{D}\,\overline{\Psi}) + 4i\mu\,\partial^{\underline{a}}\overline{D}_{\dot\alpha}\Psi_\alpha\,. & \text{(A.5)}\end{aligned}$$

Since the first term on the right is real and linear, we further obtain

$$\mu\,D^\alpha\overline{D}^2\Psi_\alpha = \mu^*\,\overline{D}_{\dot\alpha}D^2\overline{\Psi}^{\dot\alpha}\,, \tag{A.6}$$

$$D^2\overline{D}^2(D\,\Psi + \overline{D}\,\overline{\Psi}) + 4i\mu\,\partial^{\underline{a}}D^2\overline{D}_{\dot\alpha}\Psi_\alpha = 0\,. \tag{A.7}$$

Since the operator $\overline{D}^2 D^\alpha$ annihilates chiral superfields, applying it to (A.2) and making use of (A.7), we then obtain

$$\overline{D}^2(D\,\Psi + \overline{D}\,\overline{\Psi}) = D^2(D\,\Psi + \overline{D}\,\overline{\Psi}) = 0\,. \tag{A.8}$$

Next, contracting D^α on (A.2) and making use of (A.8) gives

$$i\partial^{\underline{a}}(\overline{D}_{\dot\alpha}\Psi_\alpha + D_\alpha\overline{\Psi}_{\dot\alpha}) + \mu\, D\,\Psi = 0\,. \tag{A.9}$$

We also note that, due to (A.8), the equation (A.6) is now equivalent to $\partial^{\underline{a}}(\mu\,\overline{D}_{\dot\alpha}\Psi_\alpha - \mu^*\,D_\alpha\overline{\Psi}_{\dot\alpha}) = 0$. Therefore, with the choice $\mu = i\,m$, where m is real, we end up with

$$D\,\Psi = \overline{D}\,\overline{\Psi} = 0\,. \tag{A.10}$$

Then, eq. (A.3) becomes

$$\overline{D}^2\Psi_\alpha = 0\,. \tag{A.11}$$

Finally, the equation of motion (A.2) reduces to

$$\partial_{\underline{a}}\overline{\Psi}^{\dot\alpha} + m\,\Psi_\alpha = 0\,. \tag{A.12}$$

Eqs. (A.10)–(A.12) define an irreducible $Y = 1$ massive representation. They are equivalent to the equations of motion in the Ogievetsky-Sokatchev model (2.19).

B. Derivation of (3.15)

Let us consider an action $S = S^{(\mathrm{II})}[H,\mathcal{U}] + S_m[H,\chi]$, where $S^{(\mathrm{II})}[H,\mathcal{U}]$ is the type II supergravity action, eq. (3.14), and $S_m[H,\chi]$ stands for the mass term

$$S_m[H,\chi] = -\frac{1}{2}m^2\int d^8z\, H^{\underline{a}}H_{\underline{a}} + \frac{1}{2}\gamma m^2\int d^6z\,\chi^\alpha\chi_\alpha + \frac{1}{2}\gamma^* m^2\int d^6\bar{z}\,\bar\chi_{\dot\alpha}\bar\chi^{\dot\alpha}\,, \tag{B.1}$$

with γ a complex parameter. The latter should be determined from the requirement that the equations of motion

$$\Box\left[\Pi^T_{3/2} - 2\Pi^T_{1/2}\right]H_{\underline{a}} - m^2 H_{\underline{a}} + \frac{1}{2}[D_\alpha,\overline{D}_{\dot\alpha}]\mathcal{U} = 0\,, \tag{B.2}$$

$$\frac{1}{8}\overline{D}^2 D_\alpha[D_\beta,\overline{D}_{\dot\beta}]H^{\underline{b}} + \frac{3}{4}\overline{D}^2 D_\alpha\mathcal{U} + m^2\gamma\chi_\alpha = 0\,, \tag{B.3}$$

be equivalent to (3.1). Since $\mathcal{U}$ is linear, (B.2) implies that $H_{\underline{a}}$ is linear, $D^2 H_{\underline{a}} = 0$. It is then possible to show that $\overline{D}^{\dot\alpha}H_{\underline{a}} \propto \chi_\alpha$ on-shell. To prove this proportionality, first contract $\overline{D}^{\dot\alpha}$ on (B.2) and use the following identities:

$$\overline{D}^2 D_\beta[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}} = 2i\overline{D}^2 D^\alpha\partial_{(\alpha}{}^{\dot\alpha}H_{\beta)\dot\alpha}\,, \tag{B.4}$$

$$\Box\overline{D}^{\dot\alpha}\Pi^T_{1/2}H_{\underline{a}} = -\frac{i}{8}\overline{D}^2 D^\delta\partial_{(\alpha}{}^{\dot\beta}H_{\delta)\dot\beta} = -\frac{1}{16}\overline{D}^2 D_\beta[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}}\,, \tag{B.5}$$

to arrive at:

$$+\frac{1}{8}\overline{D}^2 D_\beta[D_\alpha,\overline{D}_{\dot\alpha}]H^{\underline{a}} + \frac{3}{4}\overline{D}^2 D_\alpha\mathcal{U} - m^2\overline{D}^{\dot\alpha}H_{\underline{a}} = 0\,. \tag{B.6}$$

Substituting the first two terms with (B.3) leads to:

$$\gamma\chi_\alpha + \overline{D}^{\dot\alpha} H_{\underline{a}} = 0\,, \tag{B.7}$$

an upon substituting for $\mathcal{U}$ in (B.3) by substituting (B.7) back in yields:

$$+\frac{1}{8}\overline{D}^2 D_\alpha[D_\beta, \overline{D}_{\dot\beta}]H^{\underline{b}} - \frac{3}{4}\frac{1}{\gamma}\overline{D}^2 D_\alpha[D^\beta \overline{D}^{\dot\beta} - \frac{\gamma}{\gamma^*}\overline{D}^{\dot\beta} D^\beta]H_{\underline{b}} + m^2\gamma\chi_\alpha = 0\,. \tag{B.8}$$

This means that χ_α will vanish if γ is real and $\gamma = 6$. Equation (B.7) implies that $H_{\underline{a}}$ is irreducible when χ_α vanishes. This means that $\Pi^T_{3/2} H_{\underline{a}} = H_{\underline{a}}$ and the Klein-Gordon equation is obtained from (B.2). We therefore obtain (3.15).

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